[IUTch III, Prop 3.7]

(gl. packet thic Frobs)

$^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF}$ $\qquad \{{}^{\alpha}\mathcal{F}\}_{\alpha\in A} \quad A \subset J$

n. capsule of $\mathcal{F}$-prime-strips

${}^{\alpha}\mathcal{F} \xrightarrow{\log} {}^{\dagger}\mathcal{F}_{\alpha}$ given

$\{\underline{q}^{j^2}\} \curvearrowright (\mathcal{F}^{\times}_{mod})_j$ $\sim \mathbb{Q}$

gives log-link

(ii) (single packet nat. Frd, $\boxtimes$)

$\alpha \in A \qquad ({}^{\dagger}\mathcal{M}^{\circledast}_{MOD})_{\alpha} := \mathrm{Im}\{({}^{\dagger}\mathcal{M}^{\circledast}_{mod})_{\alpha} \to ({}^{\dagger}\mathcal{M}^{\circledast}_{mod})_{A} \to \log({}^{A}\mathcal{F}_{V_{\mathbb{Q}}})\}$

w/ $({}^{\dagger}\mathcal{F}^{\circledast}_{mod})_{\alpha} \xrightarrow{\sim} ({}^{\dagger}\mathcal{F}^{\circledast}_{MOD})_{\alpha}$

$({}^{\dagger}\mathcal{M}^{\circledast}_{mod})_{\alpha}$

$\xrightarrow{\text{alg'm}}$ Fr'd $({}^{\dagger}\mathcal{F}^{\circledast}_{MOD})_{\alpha}$

$\sim$ $\mathbb{R}$'d $({}^{\dagger}\mathcal{F}^{\circledast\mathbb{R}}_{MOD})_{\alpha}$

(iii) Isogb packet loc. frac. ideal ⊞)

$\alpha \in A$, $\left({}^{\dagger}|\overline{M}^{\circledcirc}_{\mathrm{mod}}\right)_\alpha := \left({}^{\dagger}|\overline{M}^{\circledcirc}_{\mathrm{MOD}}\right)_\alpha$

$\left(\& \left(\mathcal{F}_{\log}({}^{\dagger}A, {}^{\alpha}\mathcal{F}_{\underline{v}})\right)^{{}^\alpha\Pi_{\underline{v}}} \left(\subset \log({}^{\dagger}A, {}^{\alpha}\mathcal{F}_{\underline{v}})\right)\right)$

Fi'd $\left({}^{\dagger}\mathcal{F}^{\circledcirc}_{\mathrm{mod}}\right)_\alpha$ w/ $\left({}^{\dagger}\mathcal{F}^{\circledcirc}_{\mathrm{mod}}\right)_\alpha \xrightarrow{\sim} \left({}^{\dagger}\mathcal{F}^{\circledcirc}_{\mathrm{mod}}\right)_\alpha$

Fi'd $\quad$ R'd $\left({}^{\dagger}\mathcal{F}^{\circledcirc\mathbb{R}}_{\mathrm{mod}}\right)_\alpha$ $\quad$ (need hol. th.) $\xrightarrow{?} \left({}^{\dagger}\mathcal{F}^{\circledcirc}_{\mathrm{MOD}}\right)_\alpha$

$\alpha)\}$

$\xrightarrow{\sim} \left({}^{\dagger}\mathcal{F}^{\circledcirc}_{\mathrm{MOD}}\right)_\alpha$

(iii) (real'd gl. LGP-Fr'ds) ⊠

$\dagger\mathcal{HT}^{\Theta} \rightsquigarrow \mathcal{F}$-prime-strip $\rightsquigarrow \mathcal{F}^{\Vdash}$-prime-strip

R'd & non-real'd

$${}^{\dagger}\mathcal{C}^{\Vdash\sim}_{j} \xrightarrow{\sim} \left({}^{\dagger}\mathcal{F}^{\circledR}_{\mathrm{mod}}\right)_{j}$$

$\uparrow$ $\mathcal{F}_{\mathrm{mod}}$

real'd

$$\left({}^{\dagger}\mathcal{F}^{\circledR}_{\mathrm{mod}}\right)_{j} \xrightarrow{\sim} \left({}^{\dagger}\mathcal{F}^{\circledR}_{\mathrm{MOD}}\right)_{j}$$

composition

$$\dagger\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}} \xrightarrow{\mathrm{forg}} {}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}} \xmapsto{\text{fnt alg'm}} \mathcal{C}^{\Vdash}_{\mathrm{LGP}}\left({}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right)$$

$$\| $$

$${}^{\dagger}\mathcal{C}^{\Vdash}_{\mathrm{LGP}}$$

global real'd LGP Fr'd

$\mathcal{F}^{\Vdash}$-p

$\mathcal{F}_{\mathrm{F}}$

$\overline{\Psi}_{\mathcal{F}_{LGP}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF})$ "$\mathfrak{C}^{\Vdash}_{mod}$"

$${}^{\dagger}\mathcal{F}^{\Vdash}_{LGP} = ({}^{\dagger}\mathfrak{C}^{\Vdash}_{LGP}, \mathrm{Prime}({}^{\dagger}\mathfrak{C}^{\Vdash}_{LGP}) \xrightarrow{\sim} \underline{\mathbb{V}}, \overset{loc.}{{}^{\dagger}\mathcal{F}^{\vdash}_{LGP}}, \overset{gl. \to loc.}{\{{}^{\dagger}\rho_{LGP,\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}}})$$

$\mathcal{F}^{\Vdash}$-prime-strip

$\& \ {}^{\dagger}\mathcal{F}^{\Vdash}_{gau} \xrightarrow{\sim} {}^{\dagger}\mathcal{F}^{\Vdash}_{LGP}$ (tautol.)

global realified LGP Frobenioid

(ii) (realified gl. ℓgp-Frobenioid) $\boxplus$

$\overline{\Psi}_{\mathcal{F}_{\ell gp}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF}) := \overline{\Psi}_{\mathcal{F}_{LGP}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF})$, ${}^{\dagger}\mathcal{F}^{\vdash}_{\ell gp} := {}^{\dagger}\mathcal{F}^{\vdash}_{LGP}$

replace $({}^{\dagger}\mathcal{F}^{\circledR}_{mod})_{\alpha} \xrightarrow{\sim} ({}^{\dagger}\mathcal{F}^{\circledR}_{MOD})_{\alpha}$

by $({}^{\dagger}\mathcal{F}^{\circledR}_{mod})_{\alpha} \xrightarrow{\sim} ({}^{\dagger}\mathcal{F}^{\circledR}_{mod})_{\alpha}$

mod MOD

loc., Hl. non loc.

$\mathcal{F}^{\vdash}_{LGP}, \{{}^{\dagger}\rho_{LGP,\underline{v}}\}_{\underline{v}\in\mathbb{V}})$

taut.)

$({}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF}), \; {}^{\dagger}\mathcal{F}^{\vdash}_{lgp} := {}^{\dagger}\mathcal{F}^{\vdash}_{LGP}$

$)_{\alpha}$

$\xrightarrow{\sim} ({}^{\dagger}\mathcal{F}^{\circledR}_{mod})_{\alpha}$

mod

$${}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF} \xrightarrow[\text{lgp}]{\sim} {}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF} \underset{\text{fct. alg.}}{\longmapsto} \mathcal{C}^{\Vdash}_{lgp}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF})$$

<u>gl. real'd lgp-Fr'd</u>

$$\| \quad {}^{\dagger}\mathcal{C}^{\Vdash}_{lgp}$$

$${}^{\dagger}\mathcal{F}^{\Vdash}_{lgp} = ({}^{\dagger}\mathcal{C}^{\Vdash}_{lgp}, \mathrm{Prime}({}^{\dagger}\mathcal{C}^{\Vdash}_{lgp}) \xrightarrow{\sim} \mathbb{V}, {}^{\dagger}\mathcal{F}^{\vdash}_{lgp}, \{{}^{\dagger}\rho_{lgp,\underline{v}}\}_{\underline{v}\in\mathbb{V}})$$

$$\text{ul} \quad {}^{\dagger}\mathcal{F}^{\Vdash}_{gau} \xrightarrow{\sim} {}^{\dagger}\mathcal{F}^{\Vdash}_{LGP} \xrightarrow{\sim} {}^{\dagger}\mathcal{F}^{\Vdash}_{lgp}$$

frac. ideal ⊞)

(v) (real'd product embeddings & non-real'd gl. Fr'd)

$$\begin{array}{ccc}
\mathcal{C}^{\Vdash}_{\mathrm{LGP}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}}) & \hookrightarrow & \prod_{j\in\mathbb{F}_l^{*}} ({}^{\dagger}\mathcal{F}^{\circledast\mathbb{R}}_{\mathrm{MOD}})_j \\
\wr\| & \circlearrowright & \wr\| \\
\mathcal{C}^{\Vdash}_{\mathrm{lgp}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}}) & \hookrightarrow & \prod_{j\in\mathbb{F}_l^{*}} ({}^{\dagger}\mathcal{F}^{\circledast\mathbb{R}}_{\mathrm{mod}})_j
\end{array}$$

"$a \in K_l^{\times}$"

$\rightsquigarrow$ "$a\mathcal{O}_{K_l}$"

frac. ideal gen. by elts in $\mathcal{F}_{\mathrm{lgp}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}}) \,{}^{\dagger}\pi_1 \cong \mathcal{C}^{\Vdash}_{\mathrm{lgp}}({}^{\dagger}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}})$

obtain obj in $\mathcal{C}^{\Vdash}_{\mathrm{LGP}}$, $\mathcal{C}^{\Vdash}_{\mathrm{lgp}}$

$\mathcal{M}$

${}^{\dagger}\mathcal{L}^{\circledcirc}$

${}^{\dagger}\mathcal{D}^{\circledast\pm}$

Frd)

$^{+}\mathcal{F}^{\circledR\mathbb{R}}_{\mathrm{MON}}|_{\underline{j}}$

!?

$(^{+}\mathcal{F}^{\circledR\mathbb{R}}_{\mathrm{mod}})_{\underline{j}}$

$\wr \,{}^{\dagger}\Pi_{\sim}$

$\mathcal{C}^{\Vdash}_{\mathrm{LGP}\ \mathrm{gp}}({}^{\dagger}\mathcal{H}\mathcal{T}^{\Theta^{\pm \mathrm{ell}}\mathrm{NF}})$

M_{Θ}

${}^{\dagger}\mathcal{L}^{\circledcirc}$

${}^{\dagger}\mathcal{D}^{\circledcirc \pm}$

[IUTch III, Def 3.8]

(i) $\prod_{j \in \mathbb{F}_l^{*}} ({}^{+}\mathcal{F}^{\circledcirc}_{\mathrm{mod}})_{j}$

Ψ

$\circledR \leftarrow\!\sim\ \Psi^{\perp}_{\mathcal{F}_{\mathrm{LGP}}}({}^{\dagger}\mathcal{H}\mathcal{T}^{\Theta^{\pm \mathrm{ell}}\mathrm{NF}})$

gen. ob

ny Φ for "$q_{\underline{j}^2}$"

$\underline{q}_{\sim}$

$\circledR \in \mathrm{Ob}({}^{\dagger}\mathcal{C}^{\Vdash}_{\mathrm{LGP}\ \mathrm{gp}})$

$\circledR$-pilot obj

${}^{\dagger}\mathcal{C}^{\Vdash}_{\triangle}$

$\triangle$

splitting monoid $\leftarrow$ $({}^{\dagger}\mathcal{C}$

$\rightsquigarrow$ gen

$\rightsquigarrow$

3.8]

$(^{\dagger}\mathcal{F}_{\mathrm{mod}})_{j}$

gen. ...

$\textcircled{1} \leftarrow \sim \underline{\Psi}^{\perp}_{\mathcal{F}_{\mathrm{env}}}(^{\dagger}\mathcal{HT}^{\Theta^{\pm \mathrm{ell}}\mathrm{NF}})$

up to tor.

"$\underline{\underline{q}}^{j^2}$"

$\mathcal{E}^{\Vdash}_{\mathrm{LGP}}$

Θ - pilot obj

$^{\dagger}\mathcal{E}^{\Vdash}_{\Delta}$ $\quad \Delta = \{0, \langle \mathbb{F}_l^{*} \rangle\}$

splitting monoid $\subset (^{\dagger}\mathcal{F}^{\Vdash}_{\Delta})$

$\leadsto$ gen. up to tor.

$\leadsto \underline{\underline{q}} \in \mathrm{ob}(^{\dagger}\mathcal{E}^{\Vdash}_{\Delta})$

q - pilot obj

"$\underline{\underline{q}}$"

(ii) $\;{}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF} \xrightarrow[\ell_{\delta}]{} {}^{\ddagger}\mathcal{HT}^{\Theta^{\pm ell}NF}$

$$\wr$$

$${}^{\ddagger}\mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{LGP,\ lgp} \;\leftarrow\; \mathcal{F}^{\Vdash\blacktriangleright\times\mu}\text{-prime-strip}$$

$\underline{\mathbb{F}}_{l}^{\rtimes\pm}$

LGP $\quad \Delta$

$\dagger\ \circ \longrightarrow \circ\ *$

$\ell_0 \uparrow$

$\ddagger\ \circ$

$${}^{*}\mathcal{HT}^{\Theta^{\pm ell}N} \longrightarrow {}^{*}\mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{\Delta}$$

full poly isom of $\mathcal{F}^{\Vdash\blacktriangleright\times\mu}$-prime-strips

$${}^{\dagger}\mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{LGP,\ lgp} \xrightarrow[\text{full poly}]{\sim} {}^{*}\mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{\Delta}$$

$\mathrm{q}^{\times\mu}_{LGP,\ lgp}$-link

... product embedding (non-real'd gl. Fr'd)

$\cdots$ -ity

$\Theta^{\times\mu}_{LGP}$ - link

write

$$ {}^{\dagger}\mathcal{HT}^{\Theta^{\pm ell}NF} \xrightarrow{\Theta^{\times\mu}_{LGP}} {}^{\ddagger}\mathcal{HT}^{\Theta^{\pm ell}NF} $$

(iii) $\{ {}^{n,m}\mathcal{HT}^{\Theta^{\pm ell}NF} \}_{n,m \in \mathbb{Z}}$

$$
\begin{array}{ccccc}
\longrightarrow & {}^{n,m+1}\mathcal{HT}^{\Theta^{\pm ell}NF} & \xrightarrow{\Theta^{\times\mu}_{LGP}} & {}^{n+1,m+1}\mathcal{HT}\cdots & \\
 & \uparrow \log & & \uparrow \log & \\
 & {}^{n,m}\mathcal{HT}\cdots & \xrightarrow{\Theta^{\times\mu}_{LGP}} & {}^{n+1,m}\mathcal{HT}\cdots & \\
 & \uparrow & & \uparrow &
\end{array}
$$

LGP: Gaussian log-$\cdots$

$\log$ gp $\cdots$

[IUTchIII, Pr$\cdots$

log-vol$\cdots$

...gp-rel $_{NF}$

LGP-Gaussian log-theta lattice

Isgp1.

[IUTchIII, Prop 3.9]

$$\text{log-vol}, \quad \mu^{\log}_{\alpha, v_{\underline{\Theta}}} : \mathcal{M}(\mathcal{I}^{\mathbb{Q}}(^{\alpha}\mathcal{F}_{v_{\underline{\Theta}}})) \to \mathbb{R}, \quad v_{\underline{\Theta}} \in \mathbb{V}_{\underline{\Theta}}^{\mathrm{non}}$$

normalized ...

$$\mu^{\log}(p_v^{\mathbb{Q}}) = \mu^{\log}(\mathbb{Q}) - \log p_v$$

$$\mu^{\log}(\mathcal{O}) = 0$$

procession $\rightsquigarrow$ normalize by average

$\rightsquigarrow$ procession normalization

$$\cdots \to {}^{n,m-1}\mathcal{HT}^{\Theta^{\pm \mathrm{ell}}NF} \xrightarrow{\Theta^{\times\mu}_{LGP},\ \mathrm{gp},\ \mathrm{lp}} {}^{n+1,m+1}\mathcal{HT} \cdots$$

$$\uparrow \log \qquad\qquad \uparrow \log$$

$$ {}^{n,m}\mathcal{HT} \cdots \xrightarrow{\Theta^{\times\mu}_{LGP},\ \mathrm{lp}} {}^{n+1,m}\mathcal{HT} \cdots$$

$$\uparrow \qquad\qquad \uparrow$$

$$\mathcal{I}^{\mathbb{Q}}({}^{\mathcal{A}}\mathcal{F}_{\mathbb{V}_0}) := \prod_{v_0 \in \mathbb{V}_0} \mathcal{I}^{\mathbb{Q}}({}^{\mathcal{A}}\mathcal{F}_{v_0}) \subseteq \underline{\log}({}^{\mathcal{A}}\mathcal{F}_{v_0})$$

$\mu^{\log}_{\mathcal{A},\mathbb{V}_0}$

vol. log-vol. $\curvearrowright \mathbb{R} \longleftarrow$ μ^{lgp} ldl

degree of

inv. under a multiple by an elt $\in (\dagger\mathcal{M}^{\circledcirc}_{\mathrm{mod}})_{\Delta}$

(product formula)

$$\begin{gathered} (\dagger\mathcal{M}^{\circledcirc}_{\mathrm{mod}})_{\Delta} \\ \curvearrowright \\ \mathcal{I}^{\mathbb{Q}}({}^{\mathcal{A}}\mathcal{F}_{\mathbb{V}_0}) \end{gathered}$$

$A \overset{\text{n.i.}}{\longrightarrow} \log(A)$

$\mu^{\log(A)} = \mu^{\log(\ldots}$

$\dagger\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}} \longrightarrow \dagger\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}}$

$A \overset{\text{n.i}}{\cong} \log(A)$

$\mu^{\log(A)} \cong \mu^{\log}(\log(A))$

$|\,\cdot\,| \leq \log(A_{\mathcal{F}_{n_0}})$

... ldl

...iple by an elt $\in ({}^{\dagger}\mathcal{M}^{\ominus}_{\text{mod}})_\alpha$

$({}^{\dagger}\mathcal{M}^{\ominus}_{\text{MOD}})_\alpha$

$\hookrightarrow \mathcal{I}^{\mathbb{Q}}(A_{\mathcal{F}_{\mathbb{N}_0}})$

[IUTch III, Prop 3.10] / gl. Kummer theory & ...

$\{{}^{n,m}\mathcal{HT}^{\Theta^{\pm ell}NF}\}_{n,m \in \mathbb{Z}}$ (GP - Gaussian ...

$\rightsquigarrow {}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm ell}NF}$ D-Θ...

(vert. coricity)

(i) (vert. coric gl. LGP Frobs & assoc. K...
lgp

$\mathcal{F}({}^{n,\circ}\varsigma)_{\varsigma})_{\ast}$

et-like
${}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm ell}NF} \longmapsto$ Kum alg'm | fixd monoid

$\alpha \in A[\,]$ | Fr'd

vert. coric | $\mathcal{F}^{lt}$-μ...

[IUTch III, Prop 3.10] (gl. Kummer theory & non-interference w/ loc. integers)

$\{ {}^{n,m}\mathcal{HT}^{\Theta^{\pm ell}NF} \}_{n,m \in \mathbb{Z}}$ LGP - Gaussian log-theta lattice

$\rightsquigarrow {}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}NF}$ $\mathcal{D}\text{-}\Theta^{\pm ell}NF\text{-}\mathcal{HT}$

(vert. coricity)

(i) (vert. coric gl. LGP Frds & assoc. Kummer theory)

lgp

$\mathfrak{F}({}^{n,\circ}\mathfrak{S})_{\sharp})_*$

et-like

${}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}NF} \mapsto$

Kcx alg'm

κ-ACJ

vert. coric

fld monoid: $\mathbb{M}^{(-)\circledcirc}_{\text{mod}}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}NF})_{\sharp}$ MOD

Fr'd: $\overline{\mathcal{F}}^{\circledcirc}_{\text{mod}}({}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm ell}NF})_{\sharp} \xrightarrow{\sim} \overline{\mathcal{F}}^{\circledcirc}_{\text{mod}}(\ldots)_{\sharp} \xrightarrow{\sim} \overline{\mathcal{F}}^{\circledcirc}_{\text{MOD}}({}^{n,\circ}\ldots)_{\sharp}$

$\mathcal{F}^{\Vdash}$-pr. str: $\mathfrak{F}^{\Vdash}({}^{n,\circ}\ldots)_{gau} \xrightarrow{\sim} \mathfrak{F}^{\Vdash}({}^{n,\circ}\ldots)_{LGP} \xrightarrow{\sim} \mathfrak{F}^{\Vdash}({}^{n,\circ}\ldots)_{lgp}$

LGP - Gaussian log-theta lattice

Kummer

Frob

ét

NF monoid $\left({}^{n,m}\,{}^{(-)\circledR}\mathbb{M}_{\substack{\text{mod}\\ \text{MOD}\\ \text{mod}}}\right)_\alpha \overset{\text{Kmm}}{\xrightarrow{\sim}} {}^{(-)\circledR}\mathbb{M}_{\substack{\text{mod}\\ \text{MOD}\\ \text{mod}}}\left({}^{n,0}\mathcal{H}\mathcal{T}^{\Theta\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right)_\alpha$

Frd $\left({}^{n,m}\mathcal{F}^{\circledR}_{\substack{\text{mod}\\ \text{MOD}\\ \text{mod}}}\right)_\alpha \overset{\text{Kmm}}{\xrightarrow{\sim}} \mathcal{F}^{\circledR}_{\substack{\text{mod}\\ \text{MOD}\\ \text{mod}}}\left({}^{n,0}\mathcal{H}\mathcal{T}^{\Theta\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right)_\alpha$

$\mathcal{F}^{\Vdash}$-pr-strip ${}^{n,m}\,{}^{\triangle}\mathcal{F}^{\Vdash}_{\substack{\text{gau}\\ \text{LGP}\\ \log}} \overset{\text{Kmm}}{\xrightarrow{\sim}} {}^{\triangle}\mathcal{F}^{\Vdash}\left({}^{n,0}\mathcal{H}\mathcal{T}^{\Theta\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right)_{\substack{\text{gau}\\ \text{LGP}\\ \log}}$

$$\mathcal{I}^{\circledcirc}$$

$$\{q^{\mathbb{Z}^2}\} \subseteq \Omega(F^{\times}_{\mathrm{mod}})_{j} \quad " F^{\times}_{\mathrm{mod}} \cap \prod_{n} \mathcal{O}^{\circledcirc}_{n} = \mu$$

(ii) (non-interference w/ local integers)

$$({}^{n,m}\mathbb{M}^{\circledcirc}_{\mathrm{MOD}})_{\alpha} \cap \prod_{\underline{n} \in \underline{V}} \Phi_{\log}({}^{A,\alpha;n,m}\mathcal{F}_{\underline{n}}) = ({}^{n,m}\mathbb{M}^{\circledcirc\mu}_{\mathrm{MOD}})_{\alpha}$$

$$\Big(\subseteq \prod_{\underline{n} \in \underline{V}} \mathcal{I}^{\circledcirc}({}^{A,\alpha}\mathcal{F}_{\underline{n}}) \Big)$$

$$({}^{n,m}\mathbb{M}^{\circledcirc}_{\mathrm{MOD}})_{j} \curvearrowright \tilde{\mathcal{I}}^{\circledcirc}\big(S^{\pm}_{j+1}\,\mathcal{F}({}^{n,\circ}\mathfrak{D}_{j})|_{v_0}\big)$$

via $\mathrm{Kum}\circ(\log)^{m'}_{m' \geq 0}$

totality of these actions

mutual

log

$$\mathcal{F}^{\times}_{\mathrm{mod}} \cap \prod_{m} \mathcal{O}^{\triangleright}_{m} = \mu$$

gen

$$ \mathcal{F}_{\triangle}) = ({}^{n,m}\mathcal{M}^{\circledcirc\mu}_{\mathrm{MOD}})_{\alpha}$$

$$\leq \prod_{m \in \mathbb{Z}} \mathcal{I}^{\mathbb{Q}}({}^{A,\alpha}\mathcal{F}_{\triangle})$$

$$\tilde{\mathcal{I}}^{\mathbb{Q}}(S^{\pm}_{j+1}\mathcal{F}(^{n,\circ}\mathcal{D})_{\zeta}|_{w_0})$$

via $\mathrm{Kmm} \circ (\log)^{m'}_{m' \geq 0}$

totality of these actions

mutually compat.

log-Kummer correspondence

$(m \in \mathbb{Z})$

shift inv.

(iii) (Frd th'c log-Kummer corresp.)

$$\left({}^{n,m}\mathcal{F}^{\circledcirc(\mathbb{R})}_{\mathrm{MOD}}\right|_{\alpha} \xrightarrow[\mathrm{Kmm}]{\sim} \mathcal{F}^{\circledcirc(\mathbb{R})}_{\mathrm{MOD}}\left({}^{n,\circ}\mathcal{HT}^{\Theta\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right|_{\alpha}$$

(Imc-2) mut. compat. in the above sense

$${}^{n,m}\mathcal{F}^{\Vdash\perp}_{\mathrm{LGP}} \xrightarrow[\mathrm{Kmm}]{\sim} \mathcal{F}^{\Vdash\perp}\left({}^{n,\circ}\mathcal{HT}^{\Theta\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right|_{\mathrm{LGP}}$$

(Imc-2)

MOD, LGP

~~mod, lgp~~

[IUTch III, Th 3.11] (multiradial algorithm via LGP-monoids / Frobds)

initial Θ-data $(\overline{F}/F, X_F, l, \underline{C}_K, \underline{\mathbb{V}}, \mathbb{V}^{bad}_{mod}, \underline{\epsilon})$ fix

$\rightsquigarrow \{ {}^{n,m}\mathcal{HT}^{\Theta^{\pm ell}NF} \}_{n,m \in \mathbb{Z}}$ LGP-Gaussian log-theta lattice

$\hookrightarrow {}^{n,\circ}\mathcal{HT}^{D-\Theta^{\pm ell}NF}$ vert. coric $D\Theta^{\pm ell}NF$-$\mathcal{HT}$ up to isom.

$l^{\pm} := \frac{l+1}{2}$ $\mathfrak{D}^{\vdash}$

(i') (multirad. rep'n)

$\mathrm{Prc}({}^{n,\circ}\mathfrak{S})^{\vdash}_{T}$: $\{{}^{n,\circ}\mathfrak{S}\}^{\vdash}_{0} \hookrightarrow \{{}^{n,\circ}\mathfrak{S}^{\vdash}_{0}, {}^{n,\circ}\mathfrak{S}^{\vdash}_{1}\} \hookrightarrow \cdots \hookrightarrow \{{}^{n,\circ}\mathfrak{S}^{\vdash}_{0}, \ldots, {}^{n,\circ}\mathfrak{S}^{\vdash}_{l^{\pm}}\}$

$\Lambda(\ )$

$A\Lambda(\ )$

Consider the following data

(a) $\underline{\mathbb{V}} \ni \underline{v} \mid v_{\mathbb{Q}},\ j \in |\mathbb{F}_l|$ Any. modules $\mathcal{I}({}^{\mathfrak{S}^{\pm}_{j+1}}; {}^{n,\circ}\mathfrak{D}^{\vdash}_{v_{\mathbb{Q}}}) \subset \mathcal{I}^{\mathbb{Q}}({}^{\mathfrak{S}^{\pm}_{j+1}}; {}^{n,\circ}\mathfrak{D}^{\vdash}_{v_{\mathbb{Q}}})$

mono-an. data

w/ proc. normalized log-vol.

(h). $\underline{\mathbb{V}}^{bad} \to \underline{\mathbb{N}}$

splitting monoid

$$\overline{\mathcal{F}}^{\vdash}_{LGP}\left({}^{n,\circ}\mathcal{H}\mathcal{T}^{\Theta\text{-}\Theta^{\pm ell}NF}\right|_{\underline{\mathbb{N}}}$$

w/ action on

$$\bigcap$$

$$\prod_{j \in \mathbb{F}_l^{*}} \mathcal{I}^{\mathbb{Q}}\left(\mathcal{S}^{\pm}_{j+1}; {}^{j, n, \circ}\mathfrak{D}^{\vdash}_{\underline{\mathbb{N}}}\right)$$

via

$$\mathcal{I}^{\mathbb{Q}}\left(\cdots \mathfrak{D}^{\vdash}_{\underline{\mathbb{N}}}\right) \underset{\text{poly}}{\cong} \mathcal{I}^{\mathbb{Q}}\left(\cdots \mathcal{F}^{\vdash \times \mu}\left({}^{n,\circ}\mathcal{S}\right)_{\xi}\Big|_{\underline{\mathbb{N}}}\right) \cong \mathcal{I}^{\mathbb{Q}}\left(\cdots \mathcal{F}\left({}^{n,\circ}\mathcal{S}\right)_{\xi}\Big|_{\underline{\mathbb{N}}}\right)$$

Isomet-orbit

$\mathcal{F} \sim \mathfrak{D} \sim \mathfrak{D}^{\vdash}$

$\mathrm{Aut}^{*}(\;)$ $\quad \{q^{j^2}\} \curvearrowright \mathcal{I}^{\mathbb{Q}}$ $\quad |\mathbb{F}_l^{*}|$

(c)

$\mathcal{F} \rightsquigarrow \mathcal{D} \rightsquigarrow \mathcal{D}^{\vdash}$

$A^{\circledast}(\ )$

$\{q^{j^2}\}_j \curvearrowright \mathcal{I}^{\mathbb{Q}}$ $\mathbb{F}^{\rtimes\mu}_l$

$(\ldots \mathcal{F}({}^{n,\circ}\mathfrak{S})_{\xi})_{\succeq})$

(c). $j \in \mathbb{F}_l^{*}$

NF $\mathcal{M}^{\circledast}_{\mathrm{MOD}}({}^{n,\circ}\mathcal{H}\mathcal{T}^{\mathcal{D}\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}})_j$

$= \mathcal{M}_{\mathrm{mod}}(\ -\ )$

$\subseteq \mathcal{I}^{\mathbb{Q}}({}^{\mathbb{S}^{\pm}_{j+1};\, n,\circ}\mathcal{D}^{\vdash}_{V_0}) := \prod_{v_0 \in V_0} \mathcal{I}^{\mathbb{Q}}({}^{\mathbb{S}^{\pm}_{j+1};\, n,\circ}\mathcal{D}^{\vdash}_{v_0})$

w/ $\mathcal{F}^{\circledast(\mathbb{R})}_{\mathrm{MOD}}({}^{n,\circ}\mathcal{H}\mathcal{T}^{\mathcal{D}\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}})_j \xrightarrow{\sim} \mathcal{F}^{\circledast(\mathbb{R})}_{\mathrm{mod}}({}^{n,\circ}\mathcal{H}\mathcal{T}^{\mathcal{D}\text{-}\Theta^{\pm \mathrm{ell}}\mathrm{NF}})_j$

(log-vol can be comp'd by local log-vol's)

$^{n,\circ}\mathfrak{R}^{\mathrm{LGP}}$: the above data (a), (b), (c)
up to the following indet.

(Indet $\curvearrowright$) $\quad \mathrm{Prc}({}^{n,\circ}\mathfrak{S}^{\vdash}_{\mathbb{T}})$ $\leftarrow$ up to autom.

(Indet $\rightarrow$) $\quad v_{\underline{\mathbb{Q}}} \in \mathbb{V}_{\underline{\mathbb{Q}}}^{\mathrm{non}}$ (arc omit)

$\mathcal{I}^{\mathbb{Q}}({}^{\mathfrak{S}^{\pm}_{j+1}};{}^{n,\circ}\mathcal{D}^{\vdash}_{v_{\underline{\mathbb{Q}}}})$

$\underbrace{\mathbb{Q} \ldots \mathbb{Q}}_{j+1}$

up to Isomet on each pieces indep'ly.

$(\mathbb{P}_1 \hookrightarrow \mathbb{P}_2 \hookrightarrow$

all capsu

$(\mathbb{Q}, \subset \ldots$

morph order-preserv

$\{1,\ldots,n\}$

w/ capsule-f

P_j

j

(ii) (log-Kummer corresp.)

vert-cnic

data (a), (b), (c)

up to the following indet.

$(^{n,\circ}\varsigma)^{\vdash}_{\top}$ | ← up to autom.

$\in \mathbb{V}_{\Theta}^{non}$ (arc omit)

$I^{\Theta}(\varsigma_{j,j+1}^{\pm}; {}^{n,\circ}\mathcal{D}^{\vdash}_{n_\Theta})$

$\Theta \ldots \Theta$

$j+1$

Isomet on each pieces indep'ly.

$(\mathbb{P}_1 \subset \mathbb{P}_2 \subset \cdots \subset \mathbb{P}_n)$

all capsule-full poly-morph.

$(\Theta_1 \subset \cdots \subset \Theta_m)$

morph order-preserving inj

$\iota : \{1, \ldots, n\} \subset \{1, \ldots, m\}$

w/ capsule-full poly

$\mathbb{P}_j \subset \Theta_{\iota(j)}$

$j = 1, \ldots, n$

fct alg'm

permutation symm.

Étale picture

)

$\sim$ ${}^{n,0}\mathcal{R}^{LGP}$

fct algbm

permutation symm. of
Stable picture

$\sim$ $\mathrm{Prc}({}^{n,0}\varsigma)^{\vdash}_{T} \xrightarrow[\sim]{\text{poly}} \mathrm{Prc}({}^{n',0}\varsigma)^{\vdash}_{T}$

$$ {}^{n,0}\mathcal{R}^{LGP} \underset{\text{poly}}{\xrightarrow{\sim}} {}^{n',0}\mathcal{R}^{LGP} $$

compat. w/

$$ {}^{n,0}\varsigma)^{\vdash}_{0} \underset{\text{poly}}{\xrightarrow{\sim}} {}^{n',0}\varsigma)^{\vdash}_{0} $$ ← 0-label

$\wr$

linearity

(ii) (log-Kummer corresp.)

$n, m \in \mathbb{Z}$

$$\left\{\begin{array}{l} \mathcal{K}_{mn} : \mathcal{F}_{\mathrm{cns}}({}^{n,m}\mathcal{F}_{\succ})|_{\ast} \xrightarrow{\sim} \mathcal{F}_{\mathrm{cns}}({}^{n,\circ}\mathcal{F}_{\succ})|_{\ast} \\ \mathcal{K}_{mn} : \{\pi_1^{\mathrm{rat}}({}^{n,m}\mathcal{D}^{\circledast}) \curvearrowright {}^{n,m}\mathbb{M}^{\circledast}_{\infty\kappa}\}_j \xrightarrow{\sim} \{\pi_1^{\mathrm{rat}}({}^{n,\circ}\mathcal{D}^{\circledast}) \curvearrowright \mathbb{M}^{\circledast}_{\infty\kappa}({}^{n,\circ}\mathcal{D}^{\circledcirc})\}_j \\ \mathcal{K}_{mn} : ({}^{n,m}\overline{\mathbb{M}}^{\circledcirc}_{\mathrm{mod}})_j \xrightarrow{\sim} \overline{\mathbb{M}}^{\circledcirc}_{\mathrm{mod}}({}^{n,\circ}\mathcal{D}^{\circledcirc})_j \end{array}\right.$$

induces

(a) $\forall \underline{v} \in \mathbb{V}^{\mathrm{bad}}, j \in |\mathbb{F}_\ell|$

$$\mathcal{I}^{(\circledcirc)}({}^{S^{\pm}_{j+1};n,m}\mathcal{F}_{\underline{v}}) \xrightarrow{\sim} \mathcal{I}^{(\circledcirc)}({}^{S^{\pm}_{j+1};n,m}\mathcal{F}^{\vdash\times\mu}_{\underline{v}}) \underset{\text{Isomet-orbit}}{\overset{\text{poly}}{\xrightarrow{\sim}}} \mathcal{I}^{(\circledcirc)}({}^{S^{\pm}_{j+1};n,\circ}\mathcal{D}^{\vdash}_{\underline{v}})$$

$\{S^{\pm}_{j+1}, j\}_{(\ )_{\underline{v}}}$ $\quad$ $\{S^{\pm}_{j+1}, j\}_{(\ )_{\underline{v}}}$ $\quad$ $\{S^{\pm}_{j+1}, j\}_{(\ )_{\underline{v}}}$

compat w/ log-vol.

$n,\circ\mathfrak{R}^{\mathrm{LGP}}$: the above data (a), (b), (c) ... the following indet.

$(\mathbb{P}_1 \subset \mathbb{P}_2 \subset \cdots \subset \mathbb{P}_n)$

all capsule-full poly-morph.

(b) $\mathbb{V}^{\mathrm{lcd}} \to \underline{\sim}$ splitting monoid

const. mult. rig.

$$\Psi^{\perp}_{\mathcal{F}_{\mathrm{LGP}}}\left({}^{n,m}\mathcal{HT}^{\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right|_{\underline{\sim}} \xrightarrow[\mathrm{Kum}]{\sim} \Psi^{\perp}_{\mathrm{LGP}}\left({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right|_{\underline{\sim}}$$

Frob.   ét

(c) $j \in \mathbb{F}_{\ell}^{\circledast}$

$F^{*}_{\mathrm{mod}} \cap \prod_{\underline{v}} \mathcal{O}_{\underline{v}} = \mu$

NF & real'd / non-real'd $m \in \mathbb{Z}$ gl. Fr'd

mutually compat. (← non-interference)

$$\left({}^{n,m}\overline{\mathbb{M}}_{\substack{\mathrm{MOD}\\ \mathrm{mod}}}\right|_{j} \xrightarrow[\mathrm{Kum}]{\sim} \overline{\mathbb{M}}^{\circledcirc}_{\substack{\mathrm{MOD}\\ \mathrm{mod}}}\left({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right|_{j}$$ ← MOD-side

$$\left({}^{n,m}\mathcal{F}^{\circledcirc(\mathbb{R})}_{\substack{\mathrm{MOD}\\ \mathrm{mod}}}\right|_{j} \xrightarrow[\mathrm{Kum}]{\sim} \mathcal{F}^{\circledcirc(\mathbb{R})}_{\substack{\mathrm{MOD}\\ \mathrm{mod}}}\left({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right|_{j}$$ } $\mathbb{R}$-version

Frob   ét (vert. univ)

$${}^{n,m}\mathcal{C}^{\Vdash}_{\substack{\mathrm{LGP}\\ \mathrm{lgp}}} \xrightarrow{\sim} \mathcal{C}^{\Vdash}_{\substack{\mathrm{LGP}\\ \mathrm{lgp}}}\left({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right) \quad \mathcal{D}^{\Vdash}\left({}^{n,\circ}\mathcal{HT}^{D\text{-}\Theta^{\pm\mathrm{ell}}\mathrm{NF}}\right)_{\substack{\mathrm{LGP}\\ \mathrm{lgp}}}$$

Frob   ét.   a part of ${}^{n,m}\mathcal{D}^{\Vdash}_{\substack{\mathrm{LGP}\\ \mathrm{lgp}}}$, We consider the above data (a) up to the following indet.

(Indet $\uparrow$) $n \in \mathbb{Z}$, isom (a) is upper semi-compat.

(iii) ($\Theta^{\times\mu}_{LGP}$-link compatibility)

Kmm's in (ii) have the following properties w.r.t. $\Theta^{\times\mu}_{LGP}$-link

(a). Kmm in (ii) $\underset{F_\ell^{\times\pm}\text{-symm, rel. to}}{\frown}$ n, m $\mathcal{F}^{\vdash\times\mu}_{\Delta} \xrightarrow{\sim} \mathcal{F}^{\vdash\times\mu}_{\Delta}({}^{n,\circ}\mathfrak{S})^{\vdash}_{\Delta})$

$\mathcal{F}^{\vdash\times\mu}_{\Delta}({}^{n,\circ}\mathfrak{S})^{\vdash}_{\Delta}) \underset{\text{full poly}}{\xrightarrow{\sim}} \mathcal{F}^{\vdash\times\mu}_{\Delta}({}^{n+1,\circ}\mathfrak{S})^{\vdash}_{\Delta})$ of $\mathcal{F}^{\vdash\times\mu}$-prime-strip

is compat. w/

orig indet.

$\mathcal{I}^{\mathbb{Q}}$

q^{j^2} $(\mathcal{F}^{\times}_{\mathrm{mod}})_j$

${}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta} \underset{\substack{\text{full}\\\text{poly}}}{\subseteq} {}^{n+1,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}$

$\Theta^{\times\mu}_{LGP}$-link

$$\begin{array}{ccc}
{}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta} & \underset{\substack{\text{full}\\\text{poly}}}{\subseteq} & {}^{n+1,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta} \\
\wr\,\mathrm{Kum} & \cap & \wr\,\mathrm{Kum} \\
\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}\left({}^{n,0}\zeta^{\vdash}_{\Delta}\right) & \underset{\substack{\text{full}\\\text{poly}}}{\subseteq} & \sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}\left({}^{n+1,0}\zeta^{\vdash}_{\Delta}\right)
\end{array}$$

$\Theta^{\times\mu}_{LGP}$-link

$\mu\left({}^{n,0}\zeta^{\vdash}_{\Delta}\right)$

of $\mathcal{F}^{\vdash\times\mu}$-prime-strip

(h). $\mathcal{F}^{\vdash}$-prime strip

$\sim \mathcal{F}^{\vdash\times\mu}$-prime-

${}^{n,m}\sigma\mathcal{F}^{\vdash\times}$

$\sigma\mathcal{F}^{\vdash\times}$

${}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\Delta}$

Kum + Δ-ident $\wr$

$\sigma\mathcal{F}^{\vdash\times\mu}_{\mathrm{env}}$

$$
\begin{tikzcd}
 & {}^{n+1,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\triangle} \arrow[d, phantom, "\wr"] & \\
{}^{m}\sigma\mathcal{F}^{\vdash\times\mu}_{\triangle} \arrow[r, "\subset", "\text{full poly}"'] \arrow[d, "\wr\ \mathrm{Kmm}"] & {}^{n+1,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\triangle} \arrow[d, "\wr\ \mathrm{Kmm}"] \\
\sigma\mathcal{F}^{\vdash\times\mu}_{\triangle}\left(\begin{smallmatrix}n,0\\ \varsigma\end{smallmatrix}\right)^{\vdash}_{\triangle}\Big| \arrow[r, "\subset", "\text{full poly}"'] & \sigma\mathcal{F}^{\vdash\times\mu}_{\triangle}\left(\begin{smallmatrix}n+1,0\\ \varsigma\end{smallmatrix}\right)^{\vdash}_{\triangle}\Big|
\end{tikzcd}
$$

(h). $\mathcal{F}^{\Vdash}$-pe strips ${}^{n,m}\sigma\mathcal{F}^{\Vdash}_{env}$, $\sigma\mathcal{F}^{\Vdash}_{env}\left(\begin{smallmatrix}n,0\\ \varsigma\end{smallmatrix}\right)_{>}\Big|$

$\sim$ $\mathcal{F}^{\vdash\times\mu}$-pe-strip ism

$$ {}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\triangle} \xrightarrow{\ \cong\ } {}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{env} $$

$$ \sigma\mathcal{F}^{\vdash\times\mu}_{\triangle}\left(\begin{smallmatrix}n,0\\ \varsigma\end{smallmatrix}\right)^{\vdash}_{\triangle}\Big| \xrightarrow{\ \cong\ } \sigma\mathcal{F}^{\vdash\times\mu}_{env}\left(\begin{smallmatrix}n,0\\ \varsigma\end{smallmatrix}\right)_{>}\Big| $$

$$
\begin{tikzcd}
{}^{n,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\triangle} \arrow[r, "\overset{\text{(5)-Mh}}{\wr}\ \subset", "\text{full poly}"'] \arrow[d, "\wr"', "\substack{\mathrm{Kmm}\\+\\ \triangle\to\mathrm{env}}"] \arrow[dr, phantom, "\cap"] & {}^{n+1,m}\sigma\mathcal{F}^{\vdash\times\mu}_{\triangle} \arrow[d, "\wr\ \triangle\ \mathrm{Kmm}+\triangle\to\mathrm{env}"] \\
\sigma\mathcal{F}^{\vdash\times\mu}_{env}\left(\begin{smallmatrix}n,0\\ \varsigma\end{smallmatrix}\right)^{\vdash}_{\triangle}\Big| \arrow[r, "\subset", "\text{full poly}"'] & \sigma\mathcal{F}^{\vdash\times\mu}_{env}\left(\begin{smallmatrix}n+1,0\\ \varsigma\end{smallmatrix}\right)^{\vdash}_{\triangle}\Big|
\end{tikzcd}
$$

(c), ${}^{n,\circ}\mathcal{HT}^{\mathcal{D}\text{-}\Theta^{\pm\mathrm{ell}}NF} \simeq \in \underline{V}^{\mathrm{bad}}$

$\leadsto {}^{n,\circ}\mathfrak{R}^{\mathrm{bad}}$

$\left\{\begin{array}{ll} (a)_1 & {}_\infty\Psi^{\perp}_{\mathrm{env}}({}^\dagger\mathfrak{S}_>)_1 \supseteq {}_\infty\Psi^{\perp}_{\mathrm{env}}({}^\dagger\mathfrak{S}_>)^{\mu}_1 \\ (b)_1 & \Pi_\mu(\mathbb{M}^\Theta_*({}^\dagger\mathcal{D}_{>,1}))\otimes\mathbb{Q}/\mathbb{Z} \xrightarrow{\sim} {}_\infty\Psi_{\mathrm{env}}({}^\dagger\mathfrak{S}_>)^\mu_1 \\ (c)_1 & \mathbb{M}^\Theta_*({}^\dagger\mathcal{D}_{>,1}) \\ (d)_1 & {}_\infty\Psi^{\perp}_{\mathrm{env}}({}^\dagger\mathfrak{S}_>)_1 \xrightarrow[\text{splitting}]{} {}_\infty\Psi_{\mathrm{env}}({}^\dagger\mathfrak{S}_>)^\mu_1 \end{array}\right.$

Kum + $\left(\mathbb{M}^\Theta_*(\Pi) \underset{\text{full poly}}{\cong} \mathbb{M}^\Theta_*(\underline{\Xi})\right)$

$${}^{n,m}\mathfrak{R}^{\mathrm{bad}} \xrightarrow[\text{poly}]{\sim} {}^{n,\circ}\mathfrak{R}^{\mathrm{bad}}$$

Frd   ét

$? \subset \infty\Phi_{ens}^{\perp}(t<?)_{>}^{\mu}$

$\simeq \| \mathrm{0or2} \simeq \infty\Phi_{ens}(t\leq ?)_{>}^{\mu}$

$\to \infty\Phi_{ens}(t\leq)_{>}^{\mu}$ splitting

ét. pic. symm.

$\,{}_{n,0}\mathfrak{R}^{bad} \xrightarrow[\text{poly}]{\sim} {}_{n+1,0}\mathfrak{R}^{bad}$

$\,{}_{n,m}\sigma T^{t\times\mu} \; \Delta \xrightarrow[\text{full poly}]{\sim} {}_{n+1,m}\sigma T^{t\times\mu} \; \Delta$

"θ^{fm}" ω-lift

compat.

(d).

$\{\pi_1^{\wedge wt}({}_{n,0}\theta^{R}) - M$

$\xrightarrow[\text{poly}]{\sim} \{$

ω-lift $\sim {}_{n,m}\sigma$

Hmm in (ii)

Frob-ét

(d),

$$\{\pi_1^{\wedge\text{-}\mathrm{ut}}({}^{n,\circ}\mathcal{D}^{\circledast}) \curvearrowright \mathbb{M}^{\circledcirc}_{\infty\kappa}({}^{n,\circ}\mathcal{D}^{\circledcirc}) \to \mathbb{M}^{\circledcirc}_{\infty\kappa}({}^{n,\circ}\mathcal{D}^{\circledcirc}_{=j}) \subseteq \mathbb{M}_{\infty\kappa\times}({}^{n,\circ}\mathcal{D}^{\circledcirc}_{=j})\}_j$$

et-pictures symm "$\mathcal{D}^{\times\mu}$"

$$\xrightarrow[\text{poly}]{\sim} \{\pi_1^{\wedge\text{-}\mathrm{ut}}({}^{n+1,\circ}\mathcal{D}^{\circledast}) \curvearrowright \mathbb{M}^{\circledast}_{\infty\kappa}({}^{n+1,\circ}\mathcal{D}^{\circledcirc})$$

$$\to \mathbb{M}^{\circledcirc}_{\infty\kappa}({}^{n,\circ}\mathcal{D}^{\circledcirc}_{=j}) \subseteq \mathbb{M}_{\infty\kappa\times}({}^{n,\circ}\mathcal{D}^{\circledcirc}_{=j})\}_j$$

Θ-lth $\sim$ ${}^{n,m}\mathcal{F}^{\vdash\times\mu}_{\mathrm{LGP}} \xrightarrow[\text{full poly}]{\sim} {}^{n+1,m}\mathcal{F}^{\vdash\times\mu}_{\Delta}$.

Thm in (ii)

Fr-ét

(comput.)

d $\xrightarrow[\text{poly}]{\sim}$ ${}^{n+1,\circ}\mathfrak{R}^{\mathrm{LGP}}$

n $\xrightarrow[\text{full poly}]{\sim}$ ${}^{n+1,m}\mathcal{F}^{\vdash\times\mu}_{\Delta}$

Θ-lth

"$\mathcal{D}^{\times\mu}$"

LGP

Dgp

-had

$\mathfrak{F}^{\vdash\times\mu}$

$\triangle$

(d)

ét-picture symm "$\Theta^{\times\mu}$"

$$\{\pi_1^{\mathrm{rat}}({}^{n,\circ}\mathcal{D}^{\circledast}) \curvearrowright \mathbb{M}^{\circledast}_{\infty\kappa}({}^{n,\circ}\mathcal{D}^{\circledcirc}) \to \mathbb{M}^{\circledcirc}_{\infty\kappa}({}^{n,\circ}\mathcal{D}^{\circledcirc}_{\underline{v}_j}) \subseteq \mathbb{M}_{\infty\kappa\times}({}^{n,\circ}\mathcal{D}^{\vdash}_{\underline{v}_j})\}_j$$

$$\xrightarrow[\text{poly}]{\sim} \{\pi_1^{\mathrm{rat}}({}^{n+1,\circ}\mathcal{D}^{\circledast}) \curvearrowright \mathbb{M}^{\circledast}_{\infty\kappa}({}^{n+1,\circ}\mathcal{D}^{\circledcirc})$$

$$\to \mathbb{M}^{\circledcirc}_{\infty\kappa}({}^{n,\circ}\mathcal{D}^{\circledcirc}_{\underline{v}_j}) \subseteq \mathbb{M}_{\infty\kappa\times}({}^{n,\circ}\mathcal{D}^{\vdash}_{\underline{v}_j})\}_j$$

⊚-Lin

$$\sim {}^{n,m}\mathfrak{F}^{\vdash\times\mu}_{\mathrm{LGP}} \xrightarrow[\text{full poly}]{\sim} {}^{n+1,m}\mathfrak{F}^{\vdash\times\mu}_{\triangle}$$

Kum in (ii)

Frd-ét

compat

$$\{\underline{q}^{j^2}\}_j \curvearrowright \mathcal{I}^{\mathbb{Q}} \curvearrowleft (\mathcal{F}^{\times}_{\mathrm{mod}})_j$$

$$\underline{q} \curvearrowright \mathcal{I}^{\mathbb{Q}} \curvearrowleft (\mathcal{F}^{\times}_{\mathrm{mod}})_j$$

up to Indet's

LGP Gp

summary of Th3.11

(i) ("protagonists")

(ii) (log-Ku

unit portion $\rightsquigarrow$ $\widetilde{\underset{\sim}{I}}$

upper semi-c

(Indet ↑

non-interferon

⊞ $\mathbb{F}_l^{\rtimes\pm}$ mal. TP $\underline{\mathcal{F}}^{\vdash}$ LGP log-lk

const, mu

compat of log-link w/ $\mathbb{F}_l^{\rtimes\pm}$-symm.

non-interfer

⊠ $\mathbb{F}_l^{*}$ NF $\overline{\mathbb{M}}_{\mathrm{mod}}$ ← Belyi cuspidalization (∃ hidden endom + pGC)

$F_{\mathrm{mod}} \cap \mathbb{F}$

Θ-$(\Theta^{\pm\mathrm{ell}}$ NF-

(i) ("protagonists")

+

LGP log-lil

log-link w/ $\mathbb{F}_l^{\rtimes\pm}$-symm.

$\mathrm{l}_{\mathrm{mod}}$ ← Belyi cusp'zation (∃ hidden endom + pGC)

(ii) (log-Kummer)

upper semi-compat. $O^\times \subset \frac{1}{2p}\log(O^\times)$
(Indet ↑) $\log(O^\times)$

non-interference
const. multi. rig. ← ell. cusp'zation (∃ hidden endom + pGC)

non-interference
$F_{\mathrm{mod}} \cap \prod O = \mu$

(iii) (compat w/ $\Theta^{\times\mu}_{LGP}$

inv. under (Ind

protected from $\hat{\mathbb{Z}}$
mono-theta cy

protected from $\hat{\mathbb{Z}}$
$\mathbb{Q}_{>0} \cap \hat{\mathbb{Z}}^\times$

$\triangle$: $\mathbb{F}_\ell^{\times\pm}$-conj. synchro

(iii) (compat w/ $\Theta^{\times\mu}_{LGP}$-link)

inv. under (Indet $\longrightarrow$)

$\hat{\mathbb{Z}}^\times$-indet ← harmless for log-vol.

protected from $\hat{\mathbb{Z}}^\times$-indet (← str. of Heisenberg gp)

mono-theta cycl. rig.

protected from $\hat{\mathbb{Z}}^\times$-indet.

$\mathbb{Q}_{>0} \cap \hat{\mathbb{Z}}^\times = \{1\}$

inv. étale picture up to (Indet)

tempered conj VS prof. conj.

($\exists$ HA ⤳ tempered
 st. labels)

Θ

log

ell. cusp'ization
(← hidden endom + pGC)

-log($\mathcal{O}^\times$)

et-picture "$\theta^{\times\mu}$"

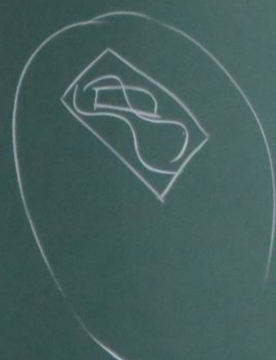
symm.

[IUTch III, Rem 3.9.5]

$\hat{\mathcal{I}}^{\circ}({}^{\alpha}\mathcal{F}_{v_\alpha})$

$\cup$

For A (rel. cpt. subset $\neq \emptyset$),



__Holomorphic hull__ of A

$\overset{\text{def}}{\iff}$ the smallest subset of $\hat{\mathcal{I}}^{\circ}(\dots)$ of the form

$$\times_{\alpha}\ \mathcal{O}_{A,\alpha}\ {}^{\alpha}\mathcal{F}_{v_\alpha}$$

$-|\log(\underline{\Theta})| \in \mathbb{R}\cup\{+\infty$

$\parallel$

proc. normalized log-vol (hol. hull of ...
the possible ...
Θ-pilot obj. ...
Kummer isom ...
rep'n of ...
w/ Indet (...

$-|\log(\underline{q})| \in \mathbb{R}$

$\parallel$

proc. normalized log-vol (the ...

summary of Th 3.11

[IUTch III, Rem 3.9.5]

$\hat{I}^{\circ}({}^{\alpha}\mathcal{F}_{\mathrm{mod}})$

(hol. cpt. subset $\neq \emptyset$),

morphic hull of A

the smallest subset of $\hat{I}^{\circ}(\dots)$ of the form

$\times \mathcal{O}_{A_{\alpha}} \, {}^{\alpha}\mathcal{F}_{\mathrm{mod}}$

$-|\log(\Theta)| \in \mathbb{R} \cup \{+\infty\}$

$\shortparallel$

proc. normalized log-vol (hol. hull of the union of the possible image of Θ-pilot obj. rel. to the multirad. Kummer isom in the multirad. rep'n of Th 3.11 w/ Indet(↑)(→)(↻))

$-|\log(\underline{q})| \in \mathbb{R}$

$\shortparallel$

proc. normalized log-vol (the image of a q-pilot obj rel. to Kummer isom in the multirad. rep'n of Th 3.11 **without** Indet(↑)(→)(↻))

[IUTch III, Cor 3.12]

$\boxed{-|\log(\underline{\underline{\Theta}})| \leq -|\log(\underline{q})|}$

"0" $\leq -|\mathrm{ht}|$

$\Updownarrow$

$\mathrm{ht} \leq (1+\epsilon)(\deg \dots$

Summary of Th 3.11

(i)("protagonists") (ii)(log-Kummer) (iii)(compat

$-|\log(\underline{\Theta})| \in \mathbb{R} \cup \{+\infty\}$

$\shortparallel$

normalized log-vol (hol. hull of the union of the possible image of Θ-pilot obj. rel. to Kummer isom in the multirad. rep'n of Th 3.11 w/ Indet $(\uparrow)(\to)(\circlearrowleft)$)

$-|\log(\underline{q})| \in \mathbb{R}$

$\shortparallel$

proc. normalized log-vol (the image of a q-pilot obj rel. to Kummer isom in the multirad. rep'n of Th 3.11 <u>without</u> Indet $(\uparrow)(\to)(\circlearrowleft)$)

[IUTch III, Cor 3.12]

$$\boxed{-|\log(\underline{q})| \leqq -|\log(\underline{\Theta})|}$$

$$\text{"}0\text{"} \leqq -(ht) + \underbrace{(\text{Indet})}_{\text{IUTch IV}}$$

$$ht \leqq (1+\epsilon)(\log\text{-diff} + \log\text{-cond})$$

HA theory ← "design" story

$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

anabelian ← tool gear

(iii) (compat w/ $\Theta^{\times\mu}_{LGP}$-link)

$\triangle$: $\mathbb{F}_\ell^{\times\pm}$-conj. synchro

indep

étale pict up to (Indet

[IUTch III, Cor 3.12]

$$\boxed{-|\log(\underline{\underline{q}})| \leq -|\log(\underline{\underline{\Theta}})|}$$

$$\text{“}0\text{''} \leq -(ht) + \underbrace{(\text{Indet})}_{\text{IUTch IV}}$$

?

$$(ht) \leq (1+\epsilon)(\log\text{-diff} + \log\text{-cond})$$

HA thy $\mapsto$ "design" story

anabelian $\mapsto$ tool geom

$$\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$$

$${}^{n,m}\mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{LGP} \xrightarrow[\text{full}]{\sim} {}^{n+1,m}\mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{\Delta}$$

$$\{\underline{\underline{q}}^{j^2}\}_{j} \stackrel{\text{poly}}{\simeq} \underline{1}^{\mathbb{N}}$$

$$\curvearrowright I \curvearrowleft \qquad \underline{\underline{q}}^{j^2} \qquad (\mathbb{F}^{\times}_{l})$$

$$\mathbb{N} \simeq \mathbb{N}$$

$$\mathcal{O}^{\times} \subset \frac{1}{2p}\log(\mathcal{O}^{\times})$$

$$\log(\mathcal{O}^{\times}) \subset$$

$\frac{1}{2}$

§.1 Reduction Steps in General Arith. Geom.

X : proper, flat scheme $/ \mathbb{Z}$
normal

$\overline{\mathcal{L}}$: arith. line bdl on X

$$ht_{\overline{\mathcal{L}}} : X(\overline{\mathbb{Q}}) \longrightarrow \mathbb{R}$$

$$x \longmapsto \frac{1}{[F:\mathbb{Q}]} \deg_F x_F^*(\overline{\mathcal{L}})$$

$X(F)$

$F/\mathbb{Q}$ fin.

$X(F) = X(\mathcal{O}_F)$

$x \longmapsto x_F$

$x \in X(F) \subseteq X$

min. fi

$\log\text{-diff}_X(x$

$x \in X(F) \subseteq X(\overline{\mathbb{Q}})$

min. field of def.

$\text{log-diff}_X(x) := \frac{1}{[F:\mathbb{Q}]} \deg_F(\delta_x) \in \mathbb{R}$

arith. div.

def'd by different $F/\mathbb{Q}$

(supported in $\mathcal{V}(F)^{\mathrm{non}}$)

$X \supset D$: (rel.) eff. Cartier div.

$U_X := X \setminus D$

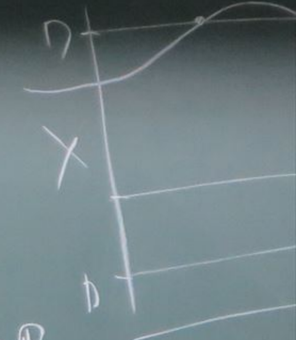
$x \in U_X(F) \subset U_X(\overline{\mathbb{Q}})$

$x_F \in X(\mathcal{O}_F)$

$\text{log-cond}_D(x) := \frac{1}{[F:\mathbb{Q}]} \deg_F(\mathfrak{f}^D_x) \in \mathbb{R}$

$\| \; (x_F^* D)_{\mathrm{red}}$

supported in $\mathcal{V}(F)^{\mathrm{non}}$



$\triangle$: $\mathbb{F}_\ell^{\times\pm}$-conj. synchro

inv

étal

up to

D: (rel.) eff. Cartier div.

$U_X := X \setminus D$

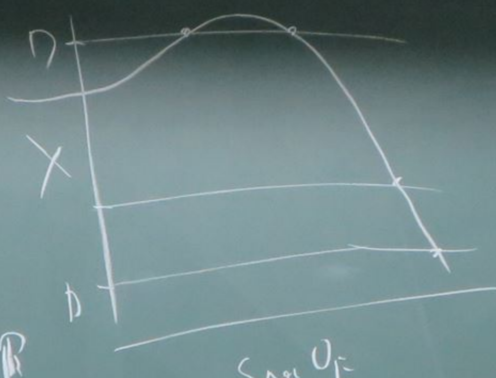
$x \in U_X(F) \subset U_X(\overline{\mathbb{Q}})$

$x_F \in X(\mathcal{O}_F)$

$\log\text{-cond}_D(x) := \frac{1}{[F:\mathbb{Q}]} \deg_F (\mathcal{F}^D_x) \in \mathbb{R}$

$= (x_F^* D)_{red}$

supported on $|\mathrm{Spec}\, \mathcal{O}_F|^{fin}$



fcts $\alpha, \beta : X(\overline{\mathbb{Q}}) \longrightarrow \mathbb{R}$

$\alpha \gtrsim \beta$ (resp. $\alpha \lesssim \beta$, $\alpha \approx \beta$)

$\overset{\text{def}}{\iff}$ $\exists C$: constant $\in \mathbb{R}$ s.t.

$\alpha(x) > \beta(x) + C$

(resp. $\alpha(x) > \beta(x) + C$, $|\alpha(x) - \beta(x)| < C$)

for $\forall x \in X(\overline{\mathbb{Q}})$

eq. class w.r.t. $\approx$: bounded discrepancy class

$\mathbb{V}_{\mathbb{Q}} \supset V$ fin. subset $\supset \mathbb{V}_{\mathbb{Q}}^{arc}$

$V \cap \mathbb{V}_{\mathbb{Q}}^{arc} \ni v$ (resp. $V \cap \mathbb{V}_{\mathbb{Q}}^{non} \ni n$)

$\phi \neq \mathcal{K}_v \subsetneq X^{arc}$ (resp. $\phi \neq K_n \subsetneq X(\overline{\mathbb{Q}}_n)$)

$X(\mathbb{C})$

a $Gal(\mathbb{C}/\mathbb{R})$-stable cpt domain

(resp. a $Gal(\overline{\mathbb{Q}}_n/\mathbb{Q}_n)$-stable subset s.t. $\forall [K:\mathbb{Q}_n] < \infty$, $X(K) \cap \mathcal{K}_n$: cpt domain)

cpt subset of top. space s.t. it is the closure of its interior

$\mathcal{K}_V \subset X(\overline{\mathbb{Q}})$ t

$\{ x \in X(\overline{\mathbb{Q}}) \mid$ $x \in X(F)$ ... $\forall n \in V \cap$ ... the set ... (resp. $X(\overline{\mathbb{Q}}$ ...

$\mathcal{K}_V$

Summary of Th 3.11

compactly bdd subset

$K_V \subset X(\bar{\mathbb{Q}})$ +

$\bigcup\!|$

$\left\{ x \in X(\bar{\mathbb{Q}}) \;\middle|\; \begin{array}{l} x \in X(F) \quad F/\mathbb{Q} \text{ fin.} \\ \forall v \in V \cap V_{\mathbb{Q}}^{ar} \text{ (resp. } v \in V \cap V_{\mathbb{Q}}^{non}) \\ \text{the set of } [F:\mathbb{Q}] \text{ pts of } X^{an} \\ \text{(resp. } X(\bar{\mathbb{Q}}_v) \text{ dof'd by } x \\ \qquad \in K_v \end{array} \right\}$

$K_V \sim V, K_v$

support.

$\bar{\mathbb{Q}}_v))$

domain

cpt subset of top. space s.t. it is the closure of its interior

Prop 1.2 ([GenEll

The following ①,

① Thm 0.1 (Vojta

X: proper

$X \supset D$:

U_X: hyperb

we ha

dd subset

ort $\supset \Sigma$

$K_v \subset X(\bar{\mathbb{Q}})$: compactly bdd subset

K_v
$\cup$
$\{ x \in X(\bar{\mathbb{Q}}) \mid x \in X(F)\ F/\mathbb{Q}$ fin. $\forall v \in V \cap V_{\mathbb{Q}}^{arc}$ (resp. $v \in V \cap V_{\mathbb{Q}}^{non}$) the set of $[F:\mathbb{Q}]$ pts of X^{an} (resp. $X(\bar{\mathbb{Q}}_v)$) det'd by x $\in K_v \}$

$K_v \sim V, K_v$ support.

$\subsetneq X(\bar{\mathbb{Q}}_v))$

st s.t.

: cpt domain

cpt subset of top. space s.t. it is the closure of its interior

Prop 1.2 ([
The follo

① Th 0.1 (
X :
$X \supset$
U_X
W

Prop 1.2 ([GenEll, Th2.1])

The following ①, ② are equiv.

① Th 2.1 (Vojta's conjecture for curves (S. Mochizuki))

X : proper smooth geom. conn. curve / an NF

$X \supset D$: red. div. $U_X := X \setminus D$, ω_X : can. sheaf on X

U_X : hyperbolic (i.e. $\deg(\omega_X(D)) > 0$) $\forall d \in \mathbb{Z}_{>0}$, $\forall \varepsilon \in \mathbb{R}_{>0}$

we have

$$ht_{\omega_X(D)} \lesssim (1+\varepsilon)(\text{log-diff}_X + \text{log-cond}_D)$$

on $U_X(\overline{\mathbb{Q}})^{\leq d}$

$$U_X(\overline{\mathbb{Q}}) = \bigcup_{F/\mathbb{Q}\,\text{fin.}} U_X(F)$$

$$\cup$$

$$U_X(\overline{\mathbb{Q}})^{\leq d} := \bigcup_{[F:\mathbb{Q}]\leq d} U_X(F)$$

fix

② Σ : fin. set of prime

$U_{\mathbb{P}^1} := \mathbb{P}^1_{\mathbb{Q}} \setminus \{0, 1, \infty\}$

$K_V \subset U_{\mathbb{P}^1}(\overline{\mathbb{Q}})$: a cpt'ly bdd subset whose support $\supset \Sigma$

$\forall d \in \mathbb{Z}_{>0}, \ \forall \varepsilon \in \mathbb{R}_{>0}$

$$ht_{\omega_{\mathbb{P}^1}(\{0,1,\infty\})} \lesssim_{\varepsilon} (1+\varepsilon)(\text{log-diff}_{\mathbb{P}^1} + \text{log-cond}_{\{0,1,\infty\}})$$

on $K_V \cap U_{\mathbb{P}^1}(\overline{\mathbb{Q}})^{\leq d}$

① $\Rightarrow$ ②

① $\Leftarrow$ ②

① $\Rightarrow$ ② special case

① $\Leftarrow$ ② non-crit. Belyi maps [Belyi] & coverings $\uparrow$! log-diff behavior of

omit

If

(omit

$+\log\text{-cond}_{\{0,1,\infty\}})$

Lem 1.3 ([IUTch IV, Prop 1.2(i)])

$k/\mathbb{Q}_p$ fin., e: ram. index of $k/\mathbb{Q}_p$

$\lambda \in \frac{1}{e}\mathbb{Z}$, $p^\lambda \mathcal{O}_k := (\varpi)^{e\lambda}$, ϖ unif

$$a := \begin{cases} \frac{1}{e}\left\lceil \frac{e}{p-2} \right\rceil & p > 2 \\ 2 & p = 2 \end{cases} \qquad b := \left\lfloor \frac{\log\left(p\frac{e}{p-1}\right)}{\log p} \right\rfloor - \frac{1}{e}$$

$$\Rightarrow \quad p^a \mathcal{O}_k \subseteq \log_p(\mathcal{O}_k^\times) \subseteq p^{-b}\mathcal{O}_k$$

log-diff. behavior of
anomaly [ ]

p [, 2 (i)])
ram. index of $k/\mathbb{Q}_p$
$\mathcal{O}_k := (\varpi)^{e\lambda}$ unit

$p>2$ $\quad h := \left[\dfrac{\log(p \frac{e}{p-1})}{\log p} \right] - \dfrac{1}{e}$
$p=2$

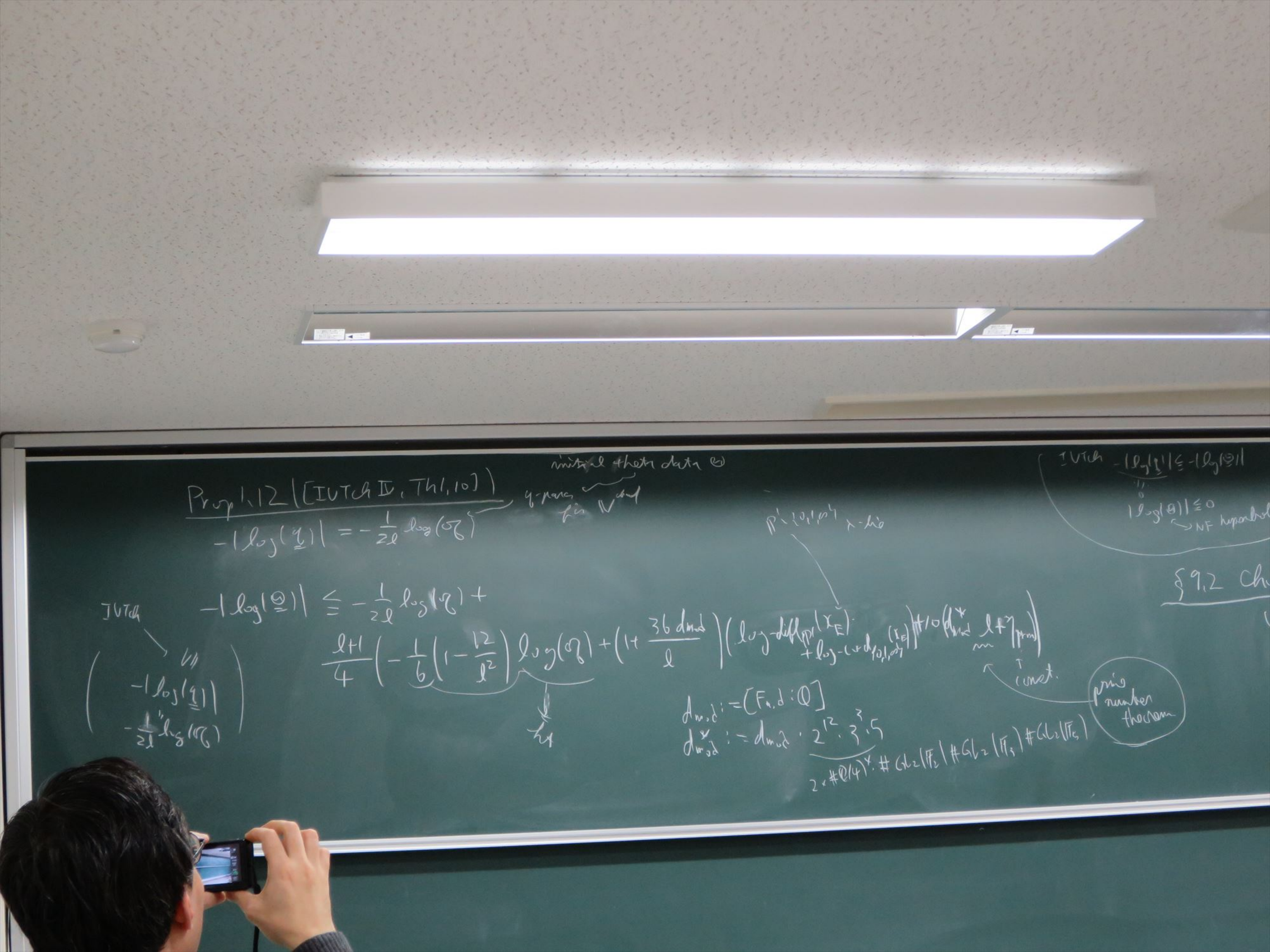
$$p^a \mathcal{O}_k \subseteq \log_p(\mathcal{O}_k^\times) \subseteq p^{-h} \mathcal{O}_k$$

If $p>2$, $e \leqq p-2$

$$\Rightarrow p^a \mathcal{O}_k = \log_p(\mathcal{O}_k^\times) = p^{-h} \mathcal{O}_k$$

(omit)

$-|\log(\circledcirc)| \leqq \circledcirc$



Prop 1.12 ([IUTch IV, Th1.10])
initial theta data ☺
q-param
fin N^{mod}
-|log(q)| = -1/(2l) log(q)
IUTch -|log(q)| ≤ -|log(Θ)|
|log(Θ)| ≤ 0
NF hyperbol
p^1 \ {0,1,∞} λ-lin
§9.2 Ch
IUTch
-|log(Θ)| ≤ -1/(2l) log(q) +
(l+1)/4 (-1/6 (1 - 12/l^2) log(q) + (1 + 36 d_mod / l) (log-diff_{P^1}(X_E) + log-cond_{0,1,∞}(X_E)) + 10 (d*_mod l + η_prm))
-|log(q)|
-1/(2l) log(q)
h_t
const.
prime number theorem
d_mod := [F_mod : Q]
d*_mod := d_mod · 2^12 · 3^3 · 5
2 × #(Z/4)^× · #GL_2(F_2) #GL_2(F_3) #GL_2(F_5)

IUTch $-|\log(\underline{q})| \leq -|\log(\Theta)|$

$|\log(\Theta)| \leq 0$

NF hyperbolicity

$\Delta \subset \mathbb{Z} \times_{\mathbb{F}_1} \mathbb{Z}$, hol. hull

$(\Delta \cdot \Delta) \leq 0$

Gauss-Bonnet

Θ-link ⟷ diff. of Frob. lifts

$(2-2g) \leq 0$

hyperbolicity

§9.2 Choice of Initial Θ-data

[IUTch IV, Cor 2.2] $U_{\mathbb{P}^1}(\overline{\mathbb{Q}}) \supset K$ cpt'ly bdd subset w/ support $\supset V_{\mathbb{Q}}^{arc}$, $2 \in V_{\mathbb{Q}}$

$U_{\mathbb{P}^1}(\overline{\mathbb{Q}}) \supset A \supset \{[E, d] \mid E$ does not admit $\overline{\mathbb{Q}}$-core$\}$ (Fact j-inv. only fin. choices)

fin. set

$\phi_2 : (\mathbb{Z}/2\mathbb{Z})^{\oplus 2} \cong E[2]$

Then $\exists C_K \in \mathbb{R}_{>0}$ s.t.

$A \in \mathbb{R}_{>0}$, $\epsilon \in \mathbb{R}_{>0}$, $d^* := 2^{12} \cdot 3^3 \cdot 5 \cdot d$

prime number theorem

$\#GL_2(\mathbb{F}_5)$

hol. hull

Gauss-Bonnet $\longrightarrow$ $(2-2g) \le 0$

hyperbolicity

(Θ)-link $\longmapsto$ diff. of Frob. lift

(Θ)-data

p'ly bdd subset w/ support $\supset \mathbb{V}_{\mathbb{Q}}^{\mathrm{arc}}$, $2 \in \mathbb{V}_{\mathbb{Q}}$

$\supset \{ [(E, d)] \mid$ E does not admit $\overline{\mathbb{Q}}$-core $\}$

fin. set

$\in \mathbb{R}_{>0}$ $\varepsilon, t,$

$\varepsilon \in \mathbb{R}_{>0}$, $d^{*} := 2^{12} \cdot 3^{3} \cdot 5 \cdot d$

Fact

j-inv.

only 4 choices

$\Downarrow$

$\mathrm{Aut}_{\overline{\mathbb{Q}}}(E_F) = \{\pm 1\}$ 0, 1728

[CanLift, Prop 2.7]

$\sim$ [Take2, Th 4.1 (i)]

$\exists$ fin. subset $\mathcal{E}xc_{K,d,\varepsilon} \subset U_{\mathbb{P}^1}(\overline{\mathbb{Q}})^{\leq d}$ s.t. & ht$_{U_{\mathbb{P}^1}(\{0,1,\infty\})}(x)$

$\mathcal{E}xc_{K,d,\varepsilon} \supset A$

$x = [(E_F, \alpha)] \in (U_{\mathbb{P}^1} \cap K) \setminus \mathcal{E}xc_{K,d,\varepsilon}$ pf)

w/ $[F:\mathbb{Q}] \leq d$

F_{mod}: field of moduli of $E_{\overline{F}} := E_F \times_F \overline{F}$

$F_{tpd} := F_{mod}(E_{F_{mod}}[2]) \subset F$

Assume all pts of $E_F[3,5]$ rat. $/F$

$\& F = F_{tpd}(\sqrt{-1}, E_{F_{tpd}}[3,5])$

$\Rightarrow$ E_F, F_{mod} arise as a part of initial Θ-data

initial theta data

P ... $[IUTch\,IV, Th1.10]$

s.t.

& $\mathrm{ht}_{\omega_{\mathbb{P}^1}(\{0,1,\infty\})}(x) \le (1+\varepsilon)\left(\begin{array}{l}\text{log-diff}_{\mathbb{P}^1}(x) \\ + \text{log-cond}_{\{0,1,\infty\}}(x)\end{array}\right) + C_K$

pf)

$\mathcal{E}xc_{K,d} := A$ ~ we will enlarge in the proof

$x = [(E_F, \alpha)] \in U_{\mathbb{P}^1}(F) \cap K \setminus \mathcal{E}xc_{K,d}$

$\vartheta(x) := \sum_{\substack{p \le x \\ \text{prime}}} \log p \sim x \quad (x \to \infty)$

(prime number thm)

(Chebyshev's ϑ-fct)

Take $\exists \xi_{\mathrm{prm}} \ge 5$ s.t.

$\frac{2}{3} x < \vartheta(x) \le \frac{4}{3} x$

for $\forall x \ge \xi_{\mathrm{prm}}$

$+C_X$

large in the proof

$\cap \mathcal{E}xc_{K,d}$

$(x \to \infty)$

...ic number thm)

$\geq \xi$ s.t.

$\vartheta(x) \leq \frac{4}{3}x$

for $\forall x \geq \xi_{prm}$

$h := h(E_{F}) = \log(\mathfrak{q}^{\forall}) = \frac{1}{[F:\mathbb{Q}]} \sum_{n \in V(F)^{non}} h_n f_n \log(p_n)$

all bad

deg. of res. ext'n $/ \mathbb{F}_{p_n}$

$\frac{1}{6} \log(\mathfrak{q}^{\forall}) \approx ht_{wpl(\{0,1,\infty\})}$

$\left(h_n = 0 \leftrightarrow E_F \text{ has good red. at } n \right)$

$\exists$ fin. many isom. classes of E_F w/ $h^{\frac{1}{2}} < \xi_{prm} + \eta_{prm}$

Enlarge $\mathcal{E}xc_{K,d}$ we may assume

(s1) $h^{\frac{1}{2}} \geq \xi_{prm} + \eta_{prm}$

(LT)-link $\leftrightarrow$ diff. of Frob. lift

$$\left(\xi_{\mathfrak{p}r m} \geqq 5,\ \eta_{\mathfrak{p}rm} > 0 \Rightarrow h^{\frac{1}{2}} \geqq 5 \right)$$

(52) $$2d^* h^{\frac{1}{2}} \log(2d^* h) \geqq 2[F:\mathbb{Q}] h^{\frac{1}{2}} \log(2[F:\mathbb{Q}] h)$$

$$\geqq \sum_{h_n \neq 0} 2 h^{-\frac{1}{2}} \log(2 h_n f_n \log(p_n)) h_n f_n \log(p_n)$$

$$\geqq \sum_{h_n \neq 0} h^{-\frac{1}{2}} \log(h_n) h_n \geqq \sum_{h_n \geqq h^{\frac{1}{2}}} h^{-\frac{1}{2}} \log(h_n) h_n \geqq \sum_{h_n \geqq h^{\frac{1}{2}}} \log(h_n)$$

(53) $$d^* h^{\frac{1}{2}} \geqq [F:\mathbb{Q}] h^{\frac{1}{2}} = \sum_{n: h_n \neq 0} h^{-\frac{1}{2}} h_n f_n \log(p_n) \geqq \sum_{n: h_n \neq 0} h^{-\frac{1}{2}} h_n \log(p_n)$$

$$\geqq \sum_{h_n \geqq h^{\frac{1}{2}}} h^{-\frac{1}{2}} h_n \log(p_n) \geqq \sum_{h_n \geqq h^{\frac{1}{2}}} \log(p_n)$$

$A \subset \text{Primes}$

$\| \{p : \text{satisfies } (S1) \text{ or } (S2) \text{ or } (S3)\}$

$(S1)$ $p \le h^{\frac{1}{2}}$

$(S2)$ $p \mid h_n \neq 0$ for some $n \in W(F)^{non}$

$(S3)$ $p = p_n$ for some $n \in W(F)^{non}$ & $h_n \geq h^{\frac{1}{2}}$

$\Rightarrow$ $(S'1)$ $\displaystyle\sum_{p:(S1)} \log(p) = \vartheta(h^{\frac{1}{2}}) \le \frac{4}{3} h^{\frac{1}{2}}$ (by 2nd $\le$ of $(s0)$ & $h^{\frac{1}{2}} \geq \xi_{p_m}$ $(\leftarrow (s1))$

$(S'2)$ $\displaystyle\sum_{\substack{p:(S2) \\ \text{not } (S3)}} \log(p) \le \sum_{h_n \geq h^{\frac{1}{2}}} \log(h_n) \le 2d^* h^{\frac{1}{2}} \log(2d^* h)$ by $(s2)$

$(S'3)$ $\displaystyle\sum_{p:(S3)} \log(p) \le d^* h^{\frac{1}{2}}$ by $(s3)$

$\Rightarrow$ (ξ'123) $\vartheta_A := \sum_{p \in A} \log(p)$

$\leq 2h^{\frac{1}{2}} + d^* h^{\frac{1}{2}} + 2d^* h^{\frac{1}{2}} \log(2d^* h)$

$\leq 4 d^* h^{\frac{1}{2}} \log(2d^* h) \leq -\xi_{prm} + 5 d^* h^{\frac{1}{2}} \log(2d^* h)$

(51)

(ξ'1), (ξ'2), (ξ'3)

$2h^{\frac{1}{2}} \leq d^* h^{\frac{1}{2}}$ & $\log(2d^* h^{\frac{1}{2}}) \geq \log 4 > 1$

$\rightsquigarrow$ $\ell \notin A$ prime s.t. $\ell \leq 2(\vartheta_A + \xi_{prm})$

($\because$ otherwise $\vartheta_A \geq \vartheta(2(\vartheta_A + \xi_{prm})) \geq \frac{2}{3}(2(\vartheta_A + \xi_{prm})) \geq \frac{4}{3}\vartheta_A$

by 2nd $\leq$ of (50)

contrad.)

$\ell \notin$

$\Rightarrow$

$\ell \notin A$

$\Rightarrow$ (P1) (upper bound of ℓ)

$(5 \leq)\ h^{\frac{1}{2}} < \ell \leq 10 d^* h^{\frac{1}{2}} \log(2d^* h) \quad (\leq 20 (d^*)^2 h^2)$

ℓ not sat. (S1) $\quad \ell \leq 2(\vartheta_A + \varepsilon_{prim})$ ℓ (S'1 2 3)

$\log(2d^* h) \leq 2d^* h \leq 2d^* h^{\frac{3}{2}} \quad \left(\begin{array}{l} \log x \leq x \\ \text{for } x \geq 1 \end{array}\right)$

(P2) (monodromy non-vanishing modulo ℓ)

$\ell \nmid h_v$ for $^\forall v \in |V(F)|^{non}$ s.t. $h_v \neq 0$

ℓ not sat. (S2)

(P3) (upper bound of monodromy at ℓ)

If $\ell = p_v$ for some $v \in |V(F)|^{non} \Rightarrow h_v < h^{\frac{1}{2}}$

ℓ: not sat. (S3)

$\ldots_A + \varepsilon_{prim})) \geq \frac{4}{3} \vartheta_A$

$\ldots$ 2nd $\leq d(S0)$

Claim 1 By enlarging $\mathrm{Exc}_{K,d}$, we may assume

(P4) $\nexists$ ℓ-cycl. subgp schemo in $E_F[\ell]$

otherwise $\exists ! \{=1$ $\frac{\ell-2}{2} h_{\mathrm{wp}^1(\{0,1,\infty\})}(x) \leq \log \ell + T_K$ $\exists$ const. $\in \mathbb{R}_{>0}$

$\leq \ell + T_K$

$\rightsquigarrow h_{\mathrm{wp}^1(\{0,1,\infty\})}(x)$: bdd

$h_{\dots}(x) \leq \frac{2\ell}{\ell-2} + \frac{2}{\ell-2} T_K$

$< \frac{14}{7-2} + \frac{2}{7-2} T_K$

$\exists$ fin. many such

non-arch part

cf. ① $\ell \cdot h_{\mathrm{wp}^1(\{0,1,\infty\})} \overset{!}{=} h^{\mathrm{non}}([E,\alpha])$

$= h^{\mathrm{non}}_{\dots}([E/H, \mathrm{Im}(\alpha)])$

ℓ-cycl.

② $2h^{\mathrm{Falt}} \lesssim h_{\mathrm{wp}^1(\{0,1,\infty\})} \lesssim 2h^{\mathrm{Falt}} + \log(h_{\mathrm{wp}^1(\{0,1,\infty\})})$

for $\forall \epsilon > 0$ on $U_{\mathbb{P}^1}(\overline{\mathbb{Q}})$ $\lesssim 2h^{\mathrm{Falt}} + \epsilon h_{\mathrm{wp}^1(\{0,1,\infty\})}$

③ $h^{\mathrm{Falt}}([E,\alpha]) - \frac{1}{2}\log \ell < h^{\mathrm{Falt}}([E/H, \mathrm{Im}(\alpha)])$

$\leq h^{\mathrm{Falt}}([E,\alpha]) + \frac{1}{2}\log \ell$

Claim 2

(P5

$\exists$ fin. many such $x = [(E, \alpha)]$'s

l; bdd

$\frac{2l}{l-2} + \frac{2}{l-2} T_k$

$\frac{14}{7-2} + \frac{2}{7-2} T_k$

Claim 2 By enlarging ε_{X}, X, d, we may assume

(P5) $\phi \neq \mathbb{V}^{\mathrm{bad}}_{\mathrm{mod}} := \{ v \in \mathbb{V}^{\mathrm{fin}}_{\mathrm{mod}} \mid v \nmid 2l,\ E_F \text{ has bad mult. red. at } v \}$

☺ (p5a) $h^{\frac{1}{2}} \log l \leq h^{\frac{1}{2}} \log(20 (d^*)^2 h^2) \leq 2 h^{\frac{1}{2}} \log(5 d^* h)$

(p5b) $\leq 8 h^{\frac{1}{2}} \log(2 (d^*)^{\frac{1}{4}} h^{\frac{1}{4}}) \leq 8 h^{\frac{1}{2}}\, 2 (d^*)^{\frac{1}{4}} h^{\frac{1}{4}} = 16 (d^*)^{\frac{1}{4}} h^{\frac{3}{4}}$

(P1) If $\mathbb{V}^{\mathrm{bad}}_{\mathrm{mod}} = \phi \Rightarrow h \approx \log(\mathfrak{q}^{\ast 2}) \underset{(P3)}{\leq} h^{\frac{1}{2}} \log l \underset{(p5b)}{\leq} 16 (d^*)^{\frac{1}{4}} h^{\frac{3}{4}}$ on $\mathcal{K}$

$\Rightarrow h^{\frac{1}{4}}$: bdd $\Rightarrow \exists$ fin. many such $x = [(E_F, \alpha)]$'s

<u>Claim 3</u> By enlarging $\mathrm{Exc}_{K,d}$, we may assume

(P6) The image of the outer hom

$$\mathrm{Gal}(\overline{\mathbb{Q}}/F) \to GL_2(\mathbb{F}_\ell)$$

$E[\ell]$

contains $SL_2(\mathbb{F}_\ell)$

(cf. [GenEll, Lem 3.1 (i), (iii)]

(P2) $\Rightarrow$ $\ell \nmid h_m \neq 0$ (P5) $\Rightarrow$ $\mathbb{V}^{\mathrm{bad}}_{\mathrm{mod}} \neq \emptyset$

the image $H \ni N_+ := \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$\vdash$ gen'ts an ℓ-Sylow subgp of $GL_2(\mathbb{F}_\ell)$

$\#\{\ell$-Sylow ...

$N_{GL_2(\mathbb{F}_\ell)}$ ...

(P4) ...

$\Rightarrow$ ...

, we may assume

m

$_2(\mathbb{F}_\ell)$

ntains $SL_2(\mathbb{F}_\ell)$

$V^{\text{bad}}_{\text{mod}} \neq \emptyset$

n ℓ-Sylow subgp S

$GL_2(\mathbb{F}_\ell)$

$\#\{\ell\text{-Sylow subgp of } GL_2(\mathbb{F}_\ell)\} = \ell + 1$

$N_{GL_2(\mathbb{F}_\ell)}(S) = \left\{\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}\right\}$

(P4) ($E[\ell] \not\simeq$ (Ragel.))

$\Rightarrow$ $H \ni$ a matrix : not upper triangular

$\Rightarrow$ $n_H := \#\{\ell\text{-Sylow in } H\} > 1$

$n_H \equiv 1 \pmod{\ell} \Rightarrow n_H = \ell + 1$ $(\Leftarrow 1 < n_H \leq \ell + 1)$

$\Rightarrow$ $N_+, N_- := \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \in H$.

$SL_2(\mathbb{F}_\ell) \underset{\text{easy}}{=} G := \langle N_+, N_- \rangle$

E, F, ℓ

(P7) $\exists \subseteq_K$, $\exists V$

$(\overline{F}/F, X_1$

(P6)

$\rightsquigarrow$ App

$= \ell + 1$

$\}$

$\left.\right)$

$\cdot$ not $\overset{\text{upper}}{\text{triangular}}$

-Sylow in $H \} > 1$

$\Rightarrow n_H = \ell + 1 \quad (\Leftarrow 1 < n_H \leq \ell + 1)$

$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \in H$.

$= \langle N_+, N_- \rangle$

$E_F, \ \overline{F}, \ \ell, \ \mathbb{V}^{\text{bad}}_{\text{mod}}$

(P7) $\exists \underline{C}_K, \ {}^{\exists}\underline{\mathbb{V}}, \ {}^{\exists}\underline{\epsilon}$ s.t.

$(\overline{F}/F, X_F, \ell, \underline{C}_K, \underline{\mathbb{V}}, \mathbb{V}^{\text{bad}}_{\text{mod}}, \underline{\epsilon})$

is an initial Θ-data

(P6)

$\rightsquigarrow$ Apply $\boxed{-|\log(\underline{\underline{q}})| \leq -|\log(\underline{\underline{\Theta}})|}$ ← IUTch

(A) $\frac{1}{b}\log(\eta) \le \left(1+\frac{80 d_{\mathrm{mod}}}{\ell}\right)\left(\log(\delta^{F+pd}) + \log(f^{F+pd})\right) + 20(d^{*}_{\mathrm{mod}}\,\ell + \eta_{\mathrm{prim}})$

$$\underset{\substack{\uparrow \\ (P1)\ \&\ 80 d_{\mathrm{mod}} < d^{*}_{\mathrm{mod}} \le d^{*}}}{\le} \left(1 + d^{*} h^{-\frac{1}{2}}\right)\left(\log(\delta^{F+pd}) + \log(f^{F+pd})\right) + 200 (d^{*})^{2} h^{\frac{1}{2}} \log(2d^{*} h) + 20\eta_{\mathrm{prim}}$$

(B) $\frac{1}{b}\log(\eta^{*2}) - \frac{1}{b}\log(\eta) \underset{\substack{\uparrow \\ (P3)\ \&\ (P5)}}{\le} \frac{1}{b} h^{\frac{1}{2}} \log \ell \underset{\substack{\uparrow \\ (P5a)}}{\le} \frac{1}{3} h^{\frac{1}{2}} \log(5d^{*} h) \underset{\substack{\uparrow \\ 5 < 2^{3}}}{\le} h^{\frac{1}{2}} \log(2d^{*} h)$

(C) $\frac{1}{b}\log(\eta^{b}) - \frac{1}{b}\log(\eta^{*2}) \overset{?}{\le} R_{\kappa}$, $\quad R_{\kappa} \in \mathbb{R}_{>0}$

prm

$h^{\frac{1}{2}}\log(2d^*h) + 20\eta_{prm}$

$\log(2d^*h)$

(A), (B), (C)

$$\frac{1}{6}h = \frac{4!!}{6}\log(\eta^*) \le \left(1+d^*h^{-\frac{1}{2}}\right)\left(\log(\delta^{F+pd}) + \log(\mathcal{F}^{F+pd}) + (15d^*)^2 h^{\frac{1}{2}}\log(2d^*h) + \frac{1}{2}C_K\right.$$

$$\le \left(1+d^*h^{-\frac{1}{2}}\right)\left(\log(\delta^{F+pd}) + \log(\mathcal{F}^{F+pd}) + \frac{1}{6}h\frac{2}{5}(60d^*)^2 h^{-\frac{1}{2}}\log(2d^*h) + \frac{1}{2}C_K\right.$$

$$C_K := 40\eta_{prm} + 2B_K \qquad 1 < \frac{1}{6}\frac{2}{5}4^2$$

$200 < 15^2$

$$\varepsilon_E := (60d^*)^2 h^{-\frac{1}{2}}\log(2d^*h) \quad \left(\ge 5d^*h^{-\frac{1}{2}}\right)$$

$$\sim (\text{Eps}) \quad \varepsilon_E \le 4(60d^*)^2 h^{-\frac{1}{2}}\log\left(2(d^*)^{\frac{1}{4}}h^{\frac{1}{4}}\right) \le 4(60d^*)^3 h^{-\frac{1}{2}}h^{\frac{1}{4}} = 4(60d^*)^3 h^{-\frac{1}{4}}$$

$(P1)$ & $8 v d_{mod} < d^*_{mod} \leq d^*$

(B) $\frac{1}{6}\log(\eta^{*2}) - \frac{1}{6}\log(\eta) \leq \frac{1}{6} h^{\frac{1}{3}} \log l \leq \frac{1}{3} h^{\frac{1}{2}} \log(5d^* h) \leq h^{\frac{1}{2}} \log(2d^* h)$

$(P3)$ & $(P5)$ $\quad (P5a) \quad 5 < 2^3$

(C) $\frac{1}{6}\log(\eta^*) - \frac{1}{6}\log(\eta^{*2}) \leq 2 R_K$, $\quad R_K \in \mathbb{R}_{>0}$

$\forall \varepsilon > 0$, If $\varepsilon_E > \min\{1, \varepsilon\}$

$\Rightarrow h^{\frac{1}{4}}$: bdd ($\leftarrow$ (Eps))

$\Rightarrow$ enlarge $Exc_{K,d}$ by an $Exc_{K,d,\varepsilon}$

we may assume $\varepsilon_E \leq \min\{1, \varepsilon\}$

$$\Rightarrow \frac{1}{6} h \leq \left(1 - \frac{2}{5}\varepsilon_E\right)^{-1}\left(1 + \frac{1}{5}\varepsilon_E\right)\left(\log|\delta^{F_{tpd}}| + \log|f^{F_{tpd}}|\right) + \left(1 - \frac{2}{5}\varepsilon_E\right)^{-1} \frac{1}{2} C_K$$

$$\leq (1+\varepsilon_E)\left(\log|\delta^{F_{tpd}}| + \log|f^{F_{tpd}}|\right) + C_K$$

$$\leq (1+\varepsilon)\left(\log\text{-}\mathrm{diff}_{\mathbb{P}^1}(x_E) + \log\text{-}\mathrm{cond}_{\{0,1,\infty\}}(x_E)\right) + C_K$$

def of ε_E: $\varepsilon \geq 5 d^* h^{-\frac{1}{2}}$

$\frac{1+\frac{1}{5}\varepsilon_E}{1-\frac{2}{5}\varepsilon_E} \leq 1 + \varepsilon_E$, $\quad 1 - \frac{2}{5}\varepsilon_E \geq \frac{1}{2}$

$\leftarrow \varepsilon_E \leq \varepsilon$, $\log\text{-}\mathrm{diff}_{\mathbb{P}^1}(x_E) = \log|\delta^{F_{tpd}}|$, $\log|f^{F_{tpd}}| \leq \log\text{-}\mathrm{cond}_{\{0,1,\infty\}}(x_E)$

$\frac{1}{6}h = \frac{1}{6}\log|\eta^{\vee}| \le \left(1+d^{*}h^{-\frac{1}{2}}\right)\left(\log|\delta^{F_{\mathrm{tpd}}}| + \log|f^{F_{\mathrm{tpd}}}|\right) + (\ldots)h\log(2d^{*}h) + \frac{1}{2}C_K$

$\le \left(1+d^{*}h^{-\frac{1}{2}}\right)\left(\log|\delta^{F_{\mathrm{tpd}}}| + \log|f^{F_{\mathrm{tpd}}}|\right) + \frac{1}{6}h\,\frac{2}{5}(60d^{*})^{2}h^{-\frac{1}{2}}\log(2d^{*}h)$

$+ \frac{1}{2}C_K$

$C_K := 40\eta_{\mathrm{prm}} + 2B_K$

$1 < \frac{1}{6}\frac{2}{5}4^{2}$

$200 < 15^{2}$

$\varepsilon_E := (60d^{*})^{2}h^{-\frac{1}{2}}\log(2d^{*}h) \quad \left(\ge 5d^{*}h^{-\frac{1}{2}}\right)$

$(\mathrm{Eps}) \quad \varepsilon_E \le 4(60d^{*})^{2}h^{-\frac{1}{2}}\log\left(2(d^{*})^{\frac{1}{4}}h^{\frac{1}{4}}\right) \le 4(60d^{*})^{3}h^{-\frac{1}{2}}h^{\frac{1}{4}}$

$= 4(60d^{*})^{3}h^{-\frac{1}{4}}$

]

$+ \frac{1}{6}\log(q^{\vee}) \approx ht_{\omega_{\mathrm{pr}}}(\{0,1,\infty\})$

$\rightarrow$ Prop. OK //

$\delta = \log\mathrm{diff} + \log\mathrm{cond}$

Riemann?

$ht \le \delta + * \delta^{\boxed{\frac{1}{2}}}\log(\delta)$

$\cap K$

1

Frob. picture $\rightsquigarrow$ étale picture

core

cart. coord. $\rightsquigarrow$ polar coord.

HA

$\sum i$

$\sum i$

~ étale picture

~ ,

~ , polar coord.

HA polar coord. "$\sqrt{\pi}$"

$$\sum j^{i}(u) \approx \frac{\ell^2}{24}[\cdot\cdot]$$

$$\sum j^2 [\log q] \approx$$

cart. coord.

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

rad. gp

log

HA polar coord. "$\sqrt{\pi}$"

$$\sum j(n) \approx \frac{\ell^2}{24} (\cdots)$$

$$\sum j^2 (\ell; q) \approx$$

cart. coord.

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

angular

log(unit)

log-shell

rad.

rad. gp

(4)-link ⟷ d(Frob. lift)

"graph of Frob."

$$1 + \xi \bar{F}_r + \frac{1}{2}\xi^2 \bar{F}_r^2$$

truncate

$$\Delta_{1,}\Delta_{1,1}$$

$$\rightsquigarrow \Delta_{1,1} \cdot (\Delta_{1,1} + \xi \Gamma_{\bar{F}r,1})$$

refine

$$= \underbrace{\Delta_{1,1} \cdot \Delta_{1,1}}_{\text{main term}} + \underbrace{\xi \Delta_{1,1} \cdot \Gamma_{Fr,1}}_{\xi \text{ term}}$$

Riem

Γ_{Fr}

R1-1 ~~Riem. zeta~~

$\frac{1}{2}$

optimal {μi rather than h-term

(1-1) ~~Mellin ξ~~

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

angular

log(unit)

log-shell

rad.

rad. gp

(u) -link $\leftrightarrow$ d(Frob. lift)

"graph of Frob."

$\Delta_{1}\cdot\Delta_{1,1}$

ϵ

$\Delta_{1,1}\cdot(\Delta_{1,1}+\epsilon\,\Gamma_{\mathrm{Fr}_{1,1}})$

refine

$= \underbrace{\Delta_{1,1}\cdot\Delta_{1,1}}_{\text{main term}} + \underbrace{\epsilon\,\Delta_{1,1}\cdot\Gamma_{\mathrm{Fr}_{1,1}}}_{\epsilon\text{ term}}$

$1+\epsilon\,\Gamma_{\mathrm{Fr}}+\frac{1}{2}\epsilon^2\Gamma_{\mathrm{Fr}}^2+\dots$

truncate

integral

Riem

Γ_{Fr}

$\frac{1}{2}$

RH ~~Riem. zeta~~

optimal $\int$ mi ... ϵ-term e

(1-1) ~~Mellin ζ~~